A Simple rule for the evolution of cooperation on graphs and social networks

Meat-eater ants feeding on a cicada, social ants cooperate and collectively gather food

PNAS: Leadership, collective behavior, and the evolution of migration

- For unstructured Natural selection favors defectors over cooperators.
- In structured population who meets whom is not random, but determined by relationship
- A simple rule for cycles , spatial lattices , random regular graphs , scale free N/W

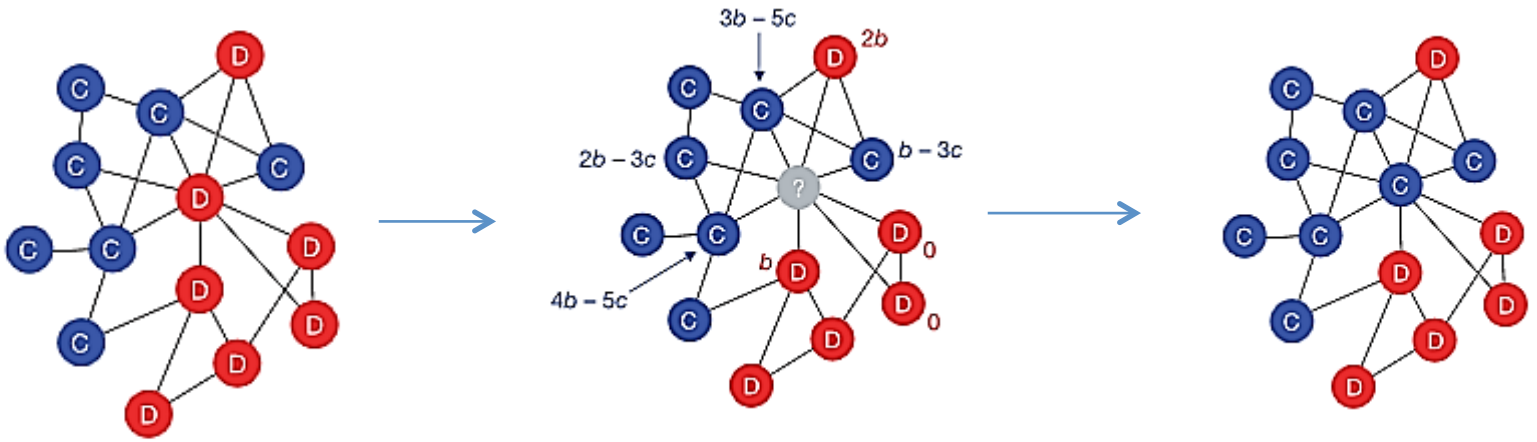
Natural Selection favors cooperators if b/c >k

Cooperators –pays a cost 'c' to receive a benefit 'b'(*Payoff=bi-ck*)

Defector- pays no cost and distribute no benefit (*Payoff=bj*)

- Fitness of individual = baseline fitness + payoff
- Strong selection = payoff > baseline fitness
- Weak selection =payoff < baseline fitness (Many different factors contribute to overall fitness of an individual)

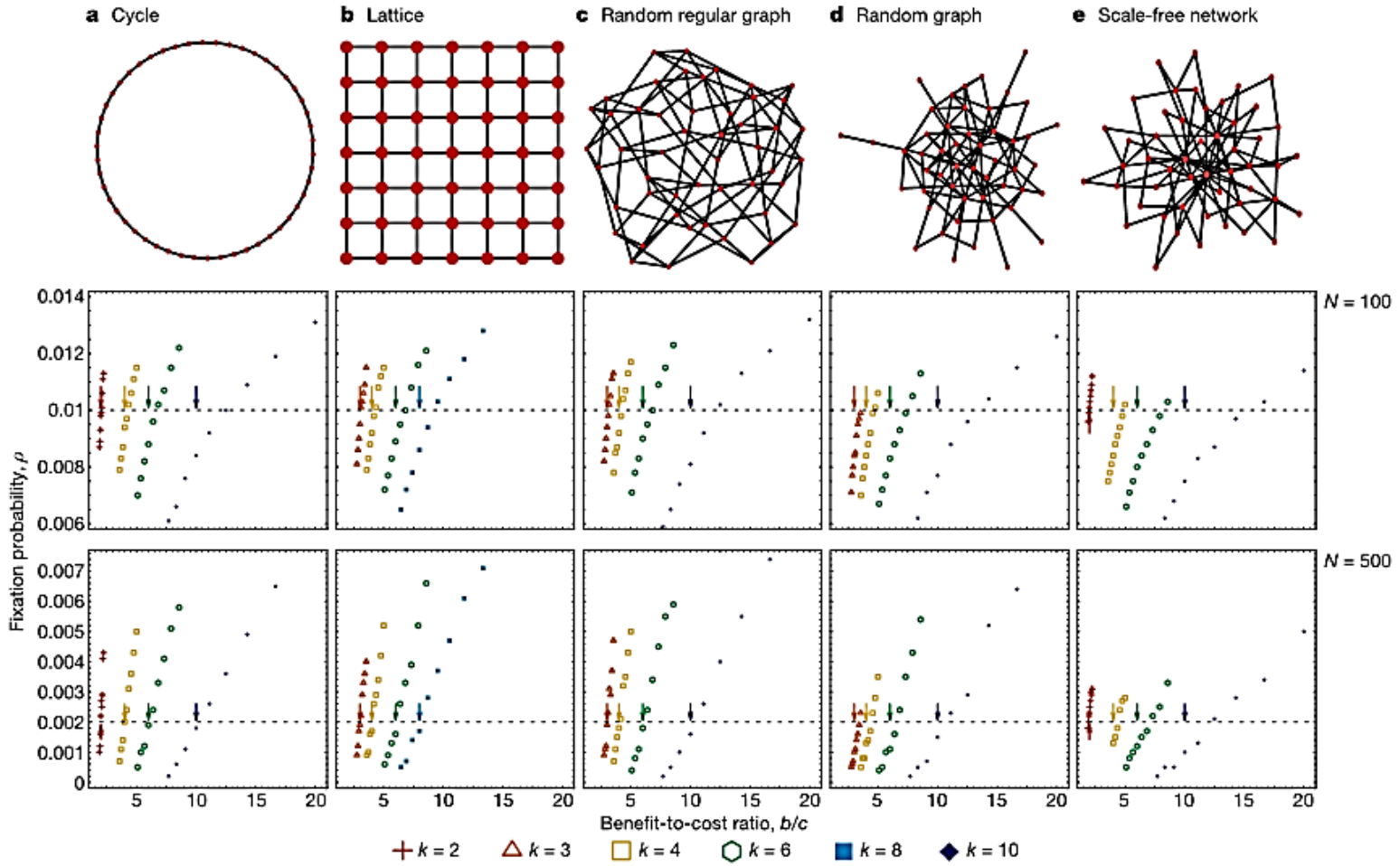
Fitness of a player =1-w+wp



We study three different update rules.

1. Death Birth
2. Imitation
3. Birthdeath

Consider '[death-birth]' (DB) updating: in each time step a random individual is chosen to die; subsequently the neighbors compete for the empty site proportional to their fitness.



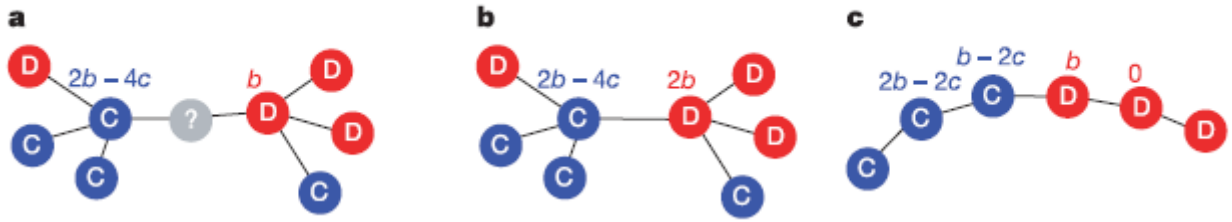
Competition arises among neighbors to take the empty site (fitness proportionality)

Fixation probability

F = 1 / N (Neutral mutant)

F > 1 / N (Selection favors emergence of cooperation)

b/c>k



The payoff for the cooperator is $P_C = bq_{C|C}(k-1) - ck.$

The payoff for the defector is $P_D = bq_{C|D}(k-1).$

Cooperation is favored if $P_C > P_D,$

From pairwise approximation for weak selection $(k-1)(q_{C|C} - q_{C|D}) = 1$

This leads to b/c>k rule $P_C - P_D = b - ck,$

2) Imitation (IM) updating: in each time step a random individual is chosen to evaluate its strategy; it will either stay with its own strategy or imitate a neighbor's strategy proportional to fitness.

b/c>k+2

3) Birth-death' (BD) updating: in each time step an individual is chosen for reproduction proportional to fitness; the offspring replaces a random neighbor. (Here only payoff at boundary matters, so selection always favors defectors)

Conclusion

- In evolutionary dynamics on graph , natural selection favors cooperation over defection if b/c>k
- Fewer connections makes easier to promote cooperation by natural selection
- Effective average degree of many networks may be small, selection of cooperation on graphs is a powerful tool.
- Average degree of graph :an inverse measure of social relatedness.

  The fewer friends I have more strongly is my fate bound to theirs.