2. Cellular Automata

Modelling Social Interaction in Information systems

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What are cellular automata (CA)?

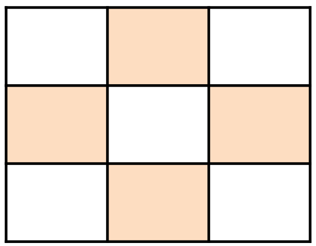
- Grid (lattice) of “cells” (2D, 1D any-D)
- Each cell can be in a finite number of states
- System evolves by applying rules that modify the state of each cell based current state + state of neighbouring cells
- Update is generally synchronous applying to all cells simultaneously in a time step
- Even with simple CA interesting dynamics can emerge

Conway’s Life

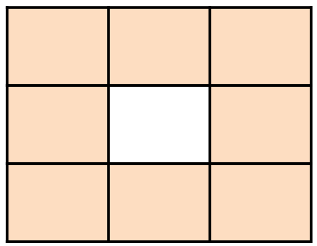
- John Conway’s “game of life” (as we saw previously) is an example of a CA in which:
  - Two cell states “alive” or “dead”
  - 2D grid
  - A neighbourhood comprising the 8 cells surrounding the cell (N,S,E,W + diagonals)
  - A simple update rule:
    - If 3 alive neighbours then cell set to alive
    - If >3 or <2 alive neighbours then cell set to dead
    - If = 2 alive neighbours then cell stays in current state

2D neighbourhoods in CA's

- Moore Neighbourhood (as in game of life)
- von Neumann Neighbourhood (just N,S,E,W)
- They can be extended to larger radius than 1
- Hence Conway's life uses "radius 1 Moore neighbourhood" which comprises 8 neighbours



von Neumann
neighbourhood



Moore
neighbourhood

Game of life

- Even with such simple rules different initial configurations can produce complex emergent behaviour
- In fact, one can produce a general purpose computer with the right initial configurations
- Golly software lets you pay with high speed CA's (including the game of life)
- NetLogo also has a life model: models library/computer science/cellular automata/life

Simple 1D CA's

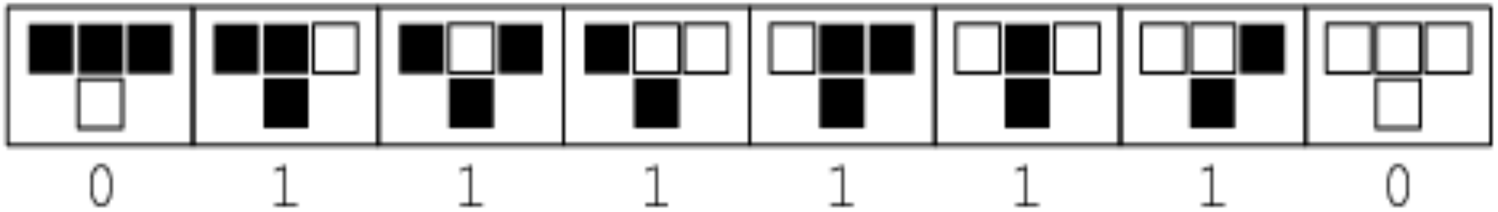
- Consider a CA with:
  - 1D grid (just a line of cells)
  - Two cell states (on or off)
  - A neighbourhood of 2 cells to left and right
- If we assume deterministic rules that map the three cell states (left, right + current cell state) to the next cell state
- There are $2^3 = 8$ different input possibilities
- Hence all possible rules can be represented by 8 bits = 2^8 = a number 0..255 (see later)

Simple 1D CA’s

- This minimal (or “Elemental”) formulation is useful because:
  - Rules can be described compactly and enumerated systematically (a number 0..255)
  - The evolution of the CA can be visualised in 2D by making one dimension time
  - Even these simple systems demonstrate the characteristics seen in more complex CA
  - A lot of work has been done with these by Stephen Wolfram (2002) and others

Simple 1D CA's

An example rule:

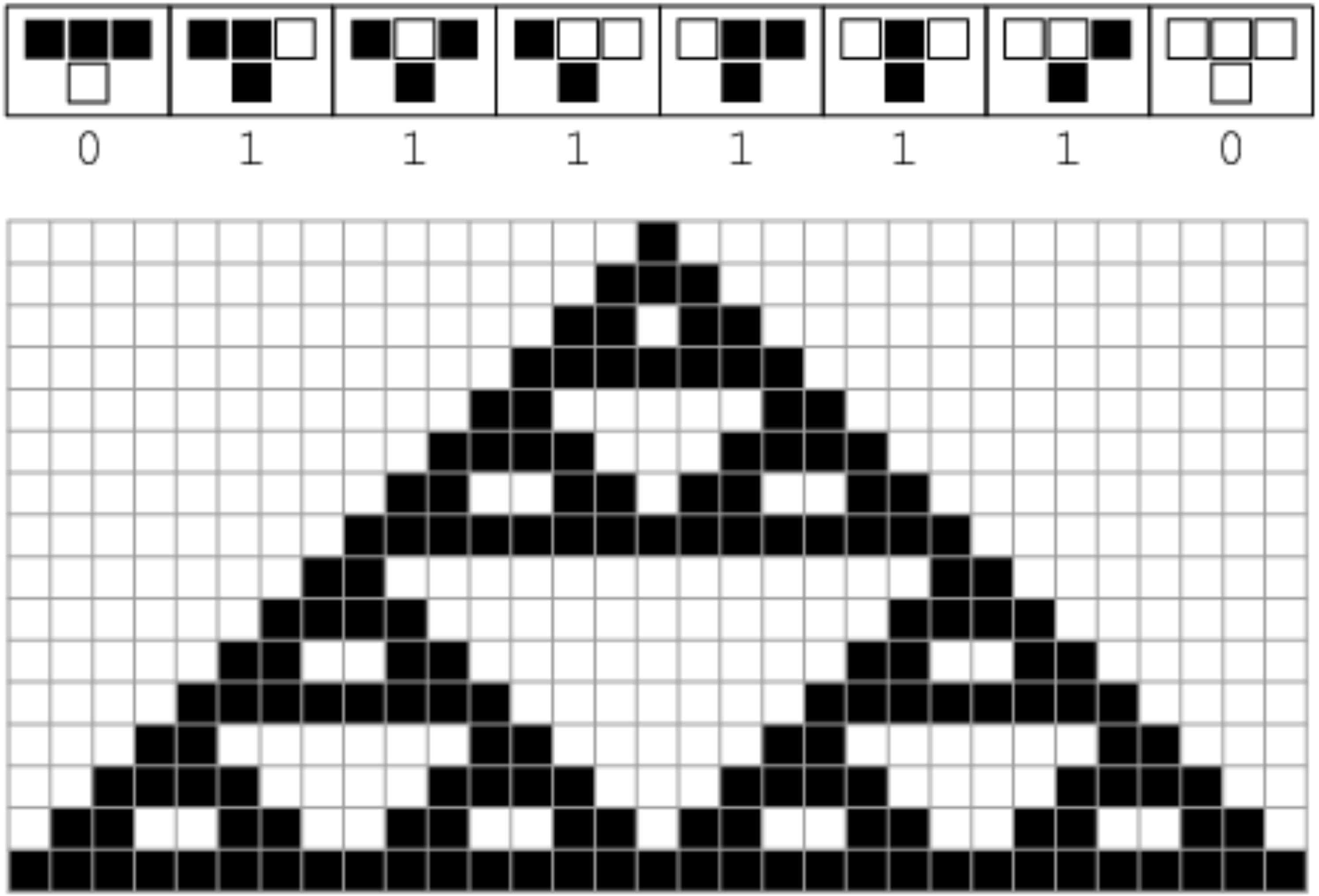


Previous	111	110	101	100	011	010	001	000
Next	0	1	1	1	1	1	1	0

Binary number for rule: 01111110 = 126 (Wolfram code)

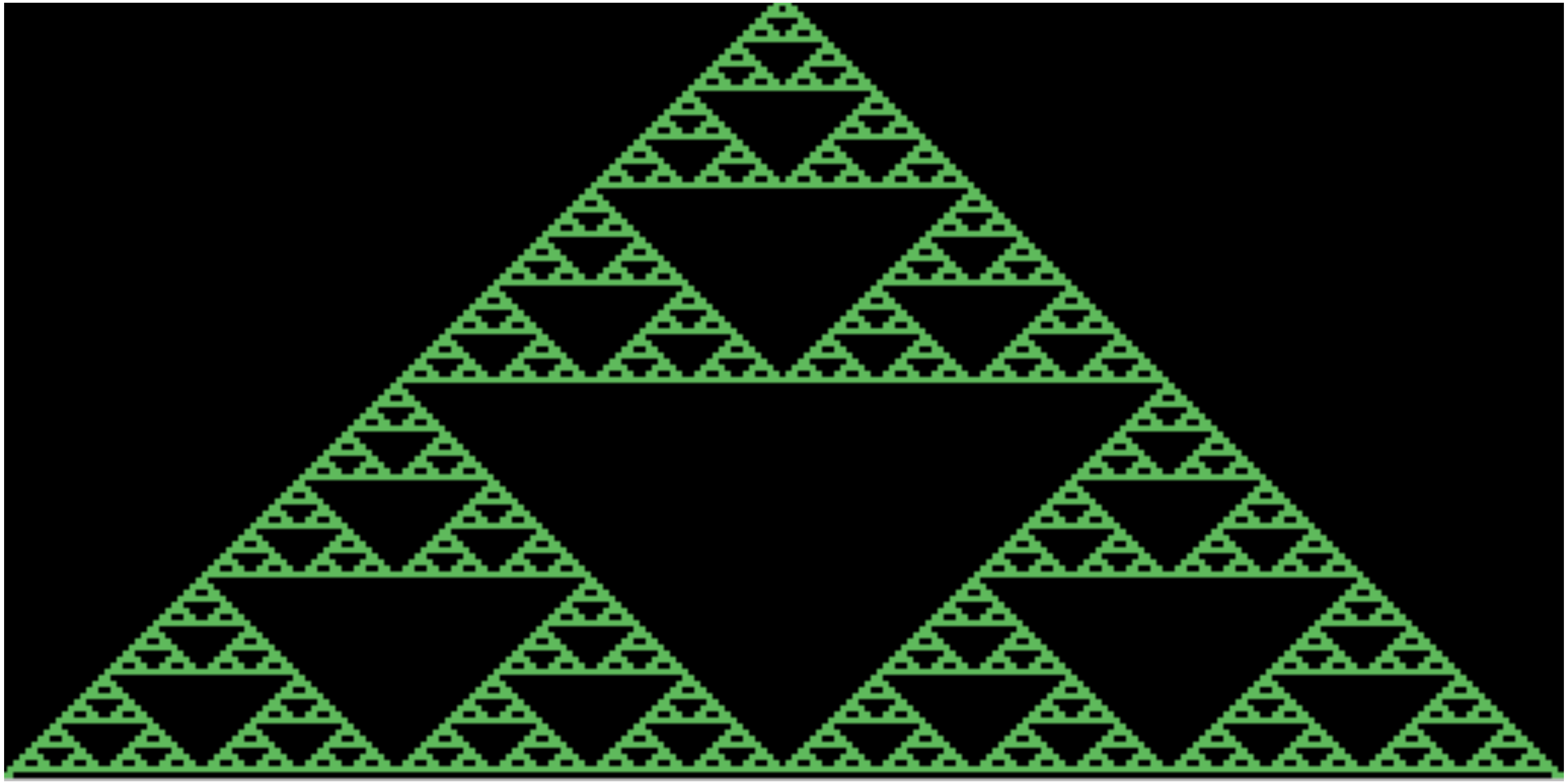
Starting with one initial seed cell set to on (1) then what will happen?

rule 126



Weisstein, Eric W. "Rule 126." From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/Rule126.html

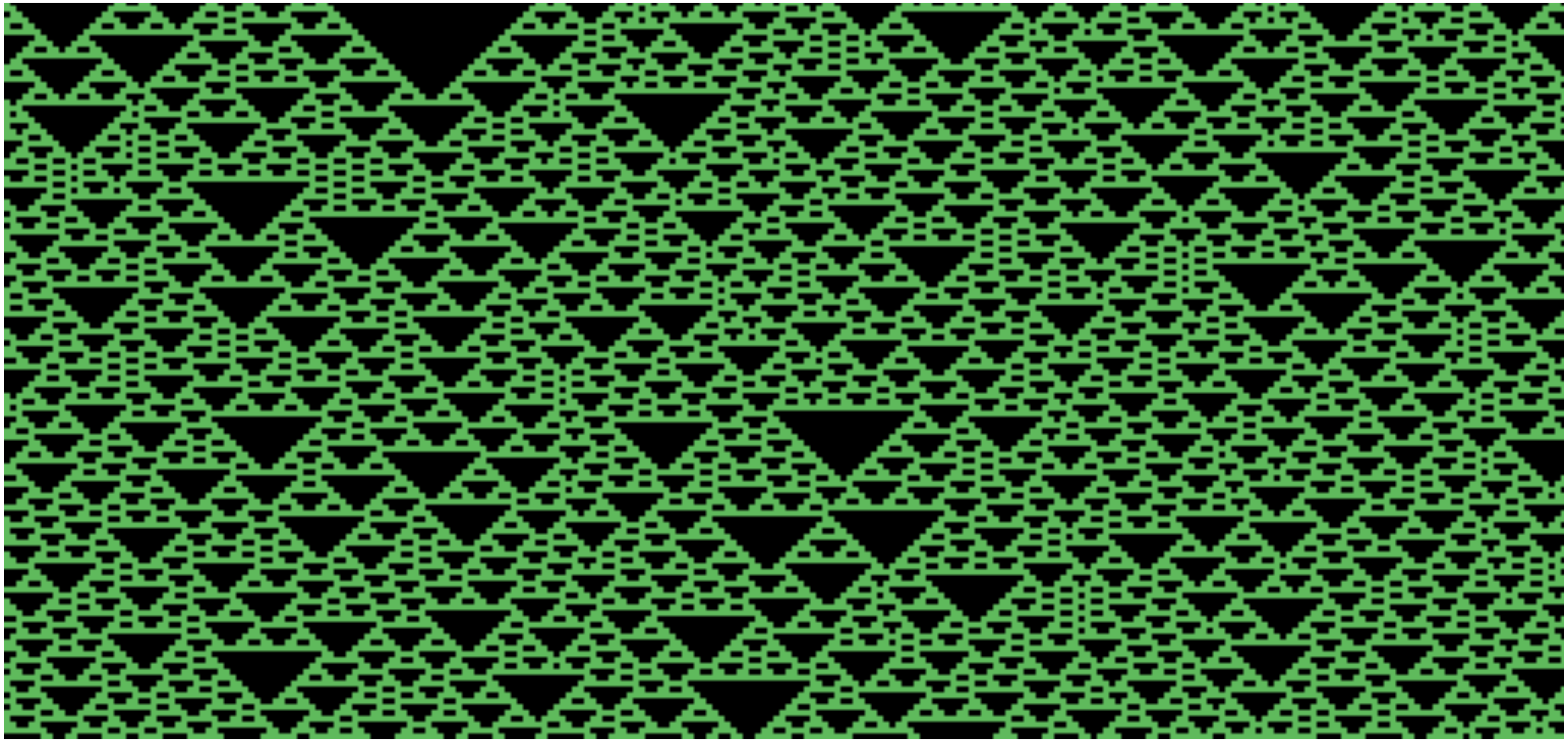
Rule 126



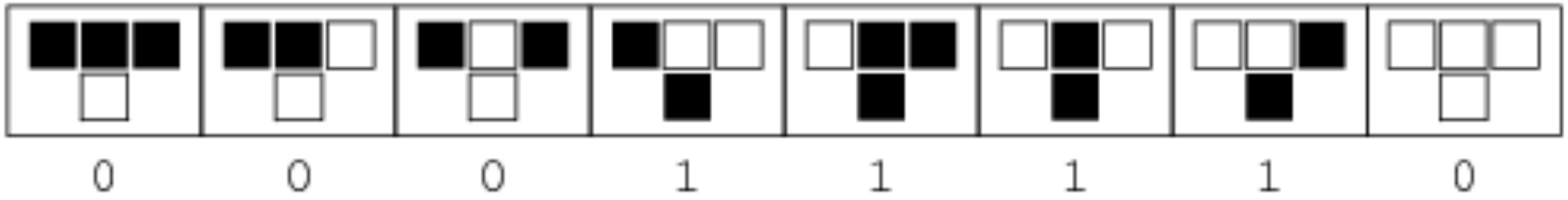
NetLogo: models library/computer science/cellular automata/CA 1D Elemental

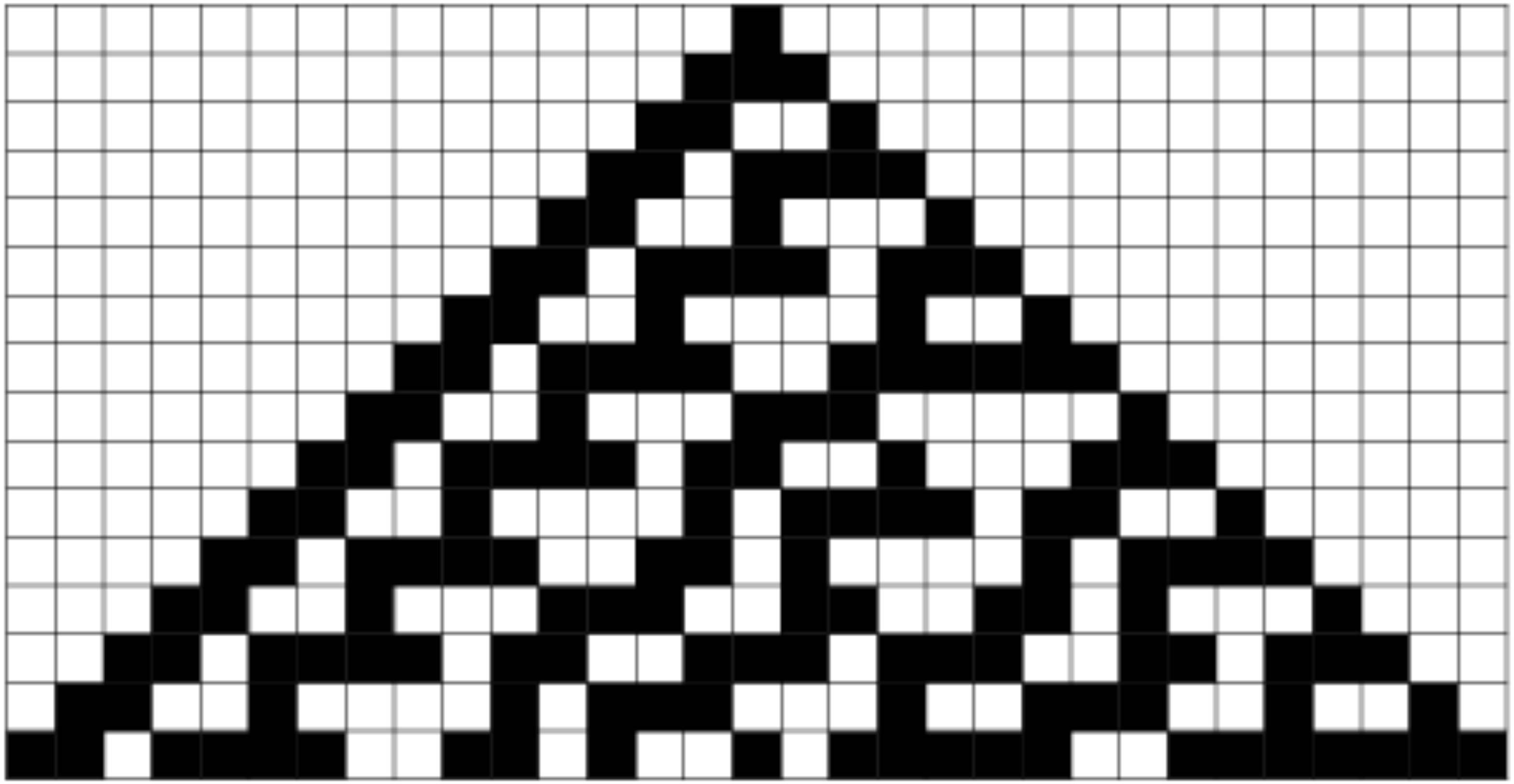
Golly: Control > newrule = W126, algorithm = hashlife

Rule 126 – random start



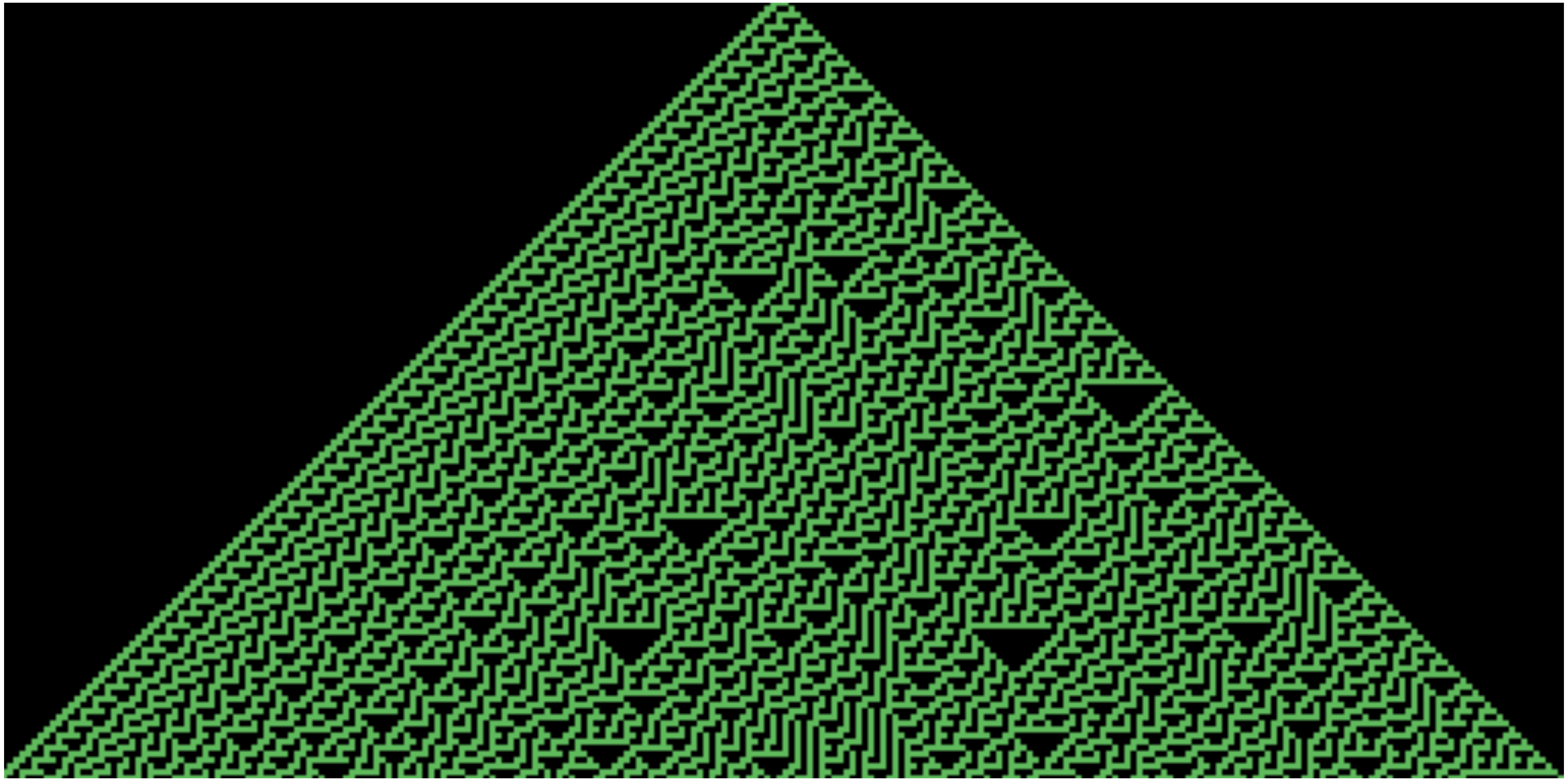
NetLogo: models library/computer science/cellular automata/CA 1D Elemental *rule 30*





Weisstein, Eric W. "Rule 30." From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/Rule30.html

Rule 30

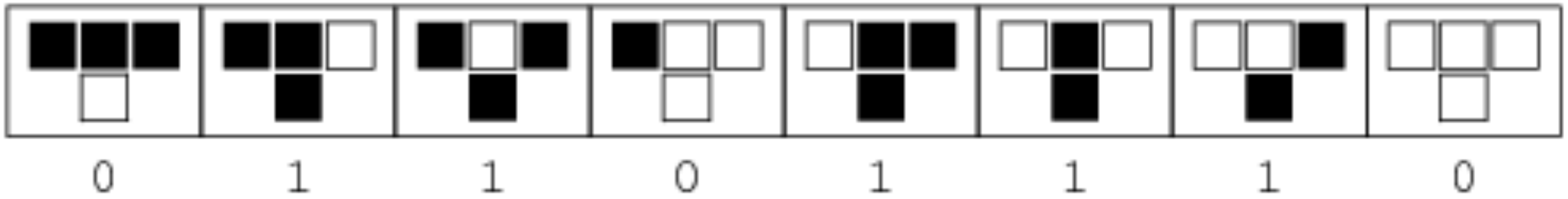


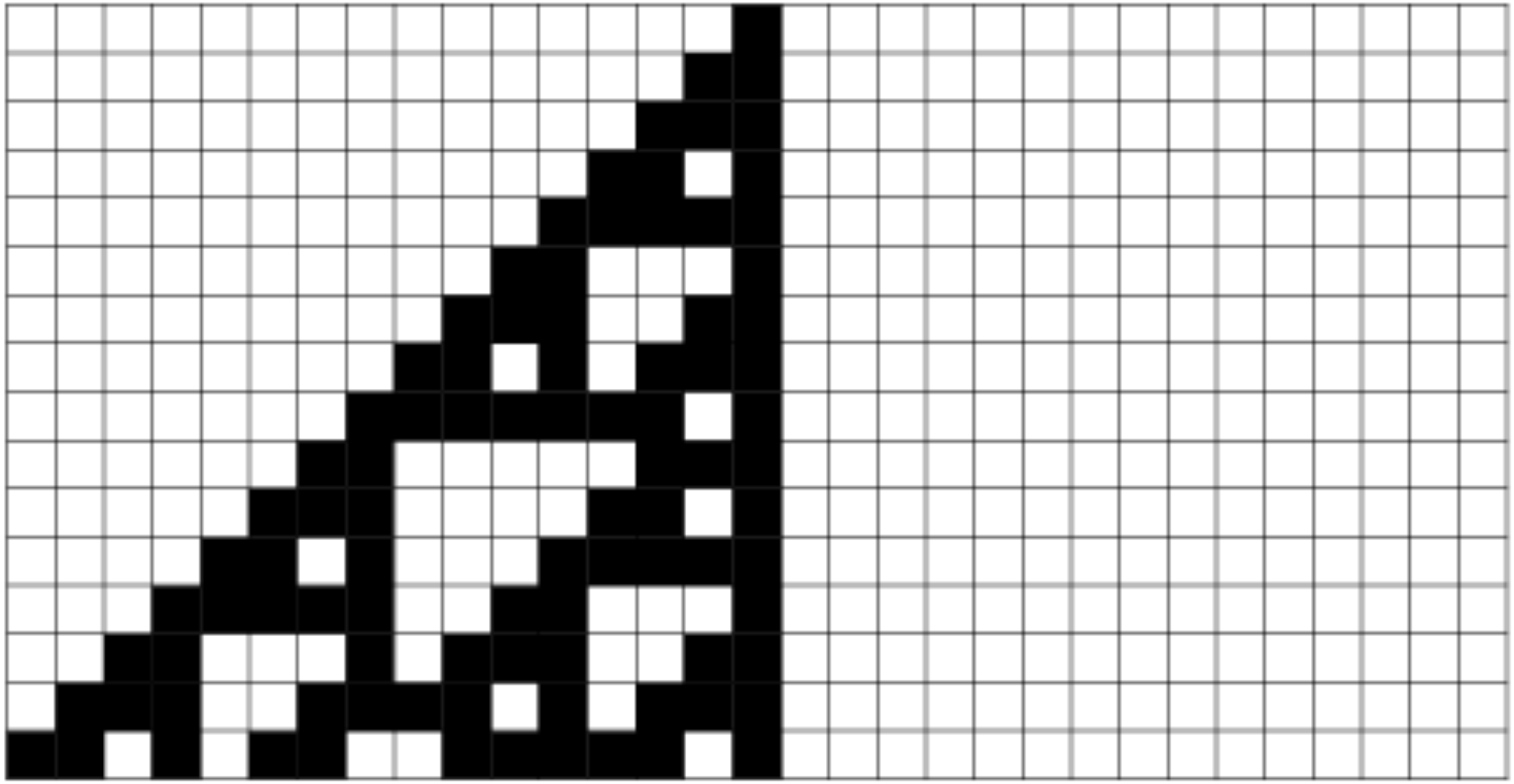
NetLogo: models library/computer science/cellular automata/CA 1D Elemental [rule 30, chaotic, used for random numbers]

Biological phenomena

Taken from: http://en.wikipedia.org/wiki/Cellular_automaton

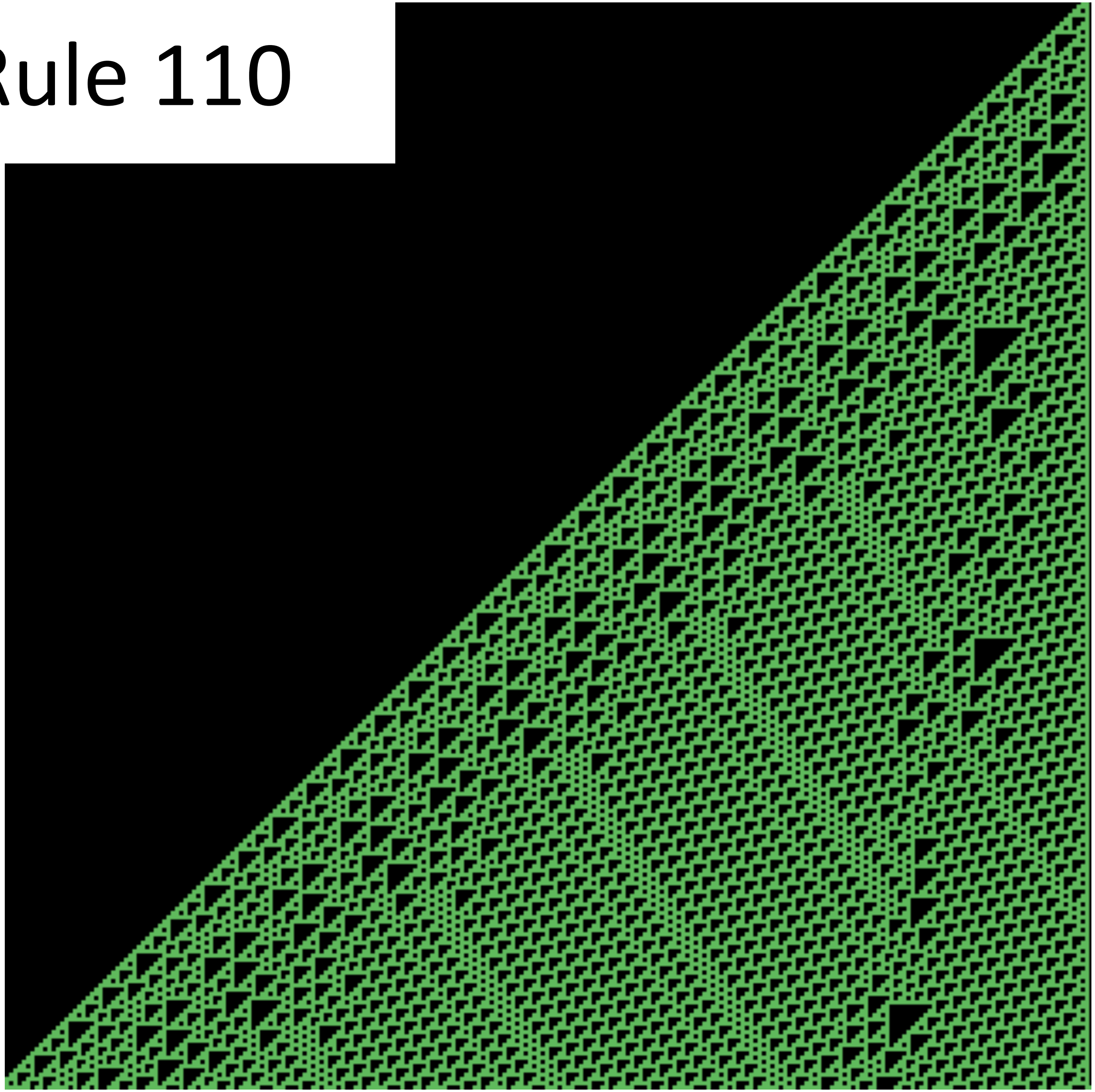
rule 110





Weisstein, Eric W. "Rule 110." From MathWorld--A Wolfram Web Resource.
http://mathworld.wolfram.com/Rule110.html

Rule 110



The infamous Rule 110 !

- So important even there was threatened legal action!
- Mathew Cook presented a proof that rule 110 was a universal computer (could simulate a Turing machine) at a conference in 1998 in Santa Fe, New Mexico
- But Wolfram (who Cook was working for) threatened legal action to suppress publication at that time
- Wolfram had worked on CA's for a very long time and had *hypothesised* universality for these kinds of rules
- Cook's proof was finally published in 2004 after the publication of Wolframs book (2002)
  - Wolfram, S. (2002) A New Kind of Science. Champaign, IL: Wolfram Media
  - Cook, M. (2004) "Universality in Elementary Cellular Automata." Complex Systems 15, 1-40, 2004.

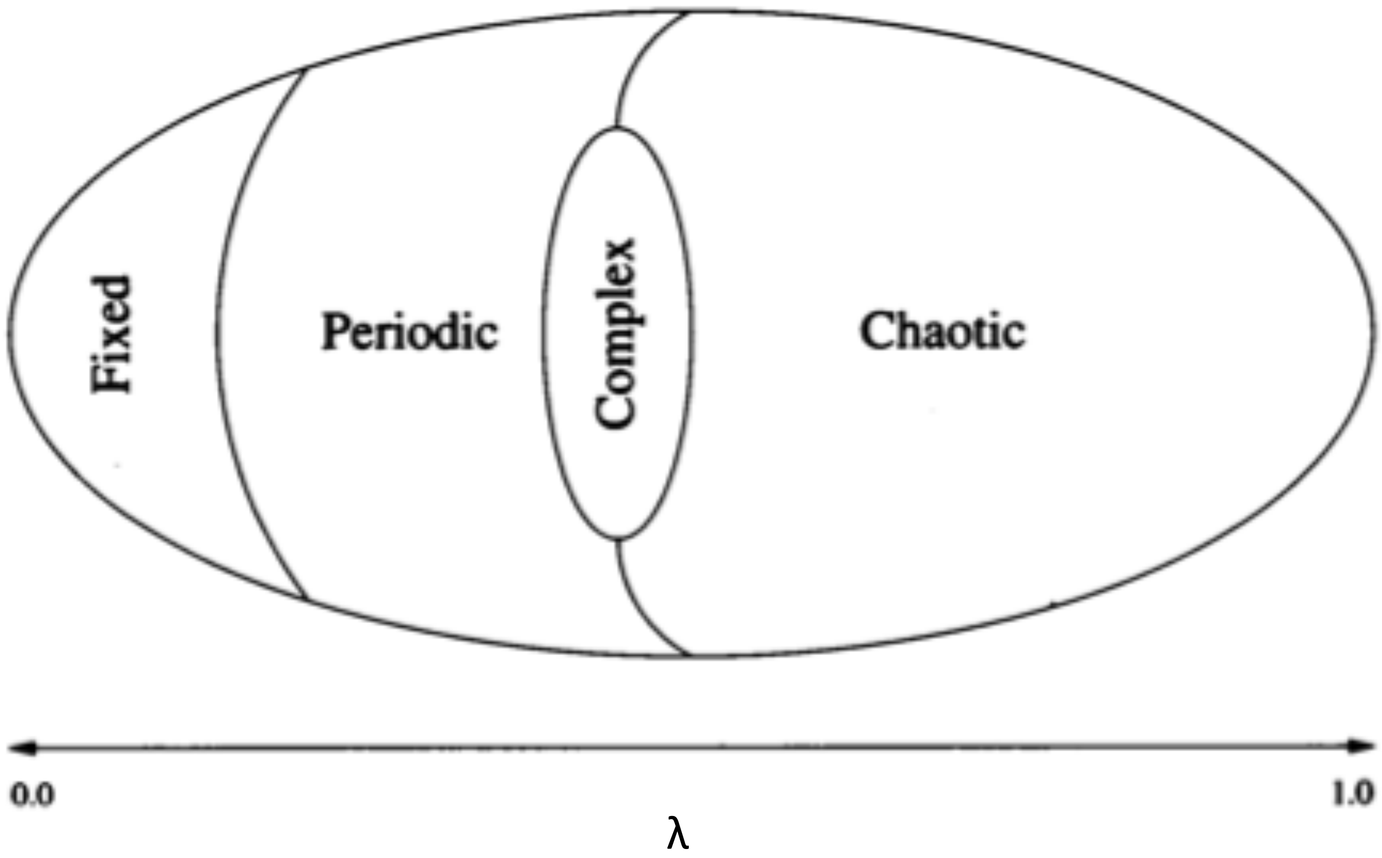
Wolframs classification

- Wolfram developed a classification for CA's:
  - Class 1: unchanging pattern
  - Class 2: oscillating patterns
  - Class 3: chaotic / random
  - Class 4: complex / interesting
- Rule 110 and the game of life are class 4
- Wolfram hypothesized that most of class 4 are universal meaning they can simulate a Turing Machine (given the right initial configurations)

Langton's Lambda (λ)

- Chris Langton came up with a way of generating a value between 0..1 for a given CA rule set
- It is the proportion of 1's in the rule set
- Values close to zero *tend* to produce unchanging patterns
- Values close to one *tend* to produce chaotic patterns
- Values between tend to produce complex patterns when they get near to chaotic
- Christopher G. Langton. "Computation at the edge of chaos". Physica D, 42, 1990.
- The idea of the "edge of chaos" has been appropriated from this work into various contexts

Langton's Lambda



Taken from: Gary Flake (1998) The Computational Beauty of Nature. MIT Press. Page 244. [compare phase transitions in physics]

Summary

- Even very simple systems in which entities with limited states interact we find:
  - Different classes of behaviour emerge
  - A class of complex and interesting behaviour
  - Such systems can even implement universal computation
  - Hence, they may be hard (impossible) to predict without simulating them
  - In fact you can not always predict what a computer program will do without running it. Otherwise why would we run it in the first place?

Summary

- We looked at simple CA's but there an many variants:
  - Many more states than two (often displayed as colours)
  - Non-deterministic CA with randomness in the rules
  - Non-discrete CA with continuous (Real) values for states
  - Migration CA in which cells can "move" on the grid
  - CA on irregular grids or networks
  - Asynchronous CA where cells are updated in a random order rather than all at the same time
- In general the further away from a simple CA a model is the less likely it is to be viewed as a CA

Summary

- What use are CA's?
  - Fun to play about with (but dangerous because one could spend a lifetime playing with them!)
  - In computing (parallel processing, random numbers, others)
  - In biology and physics (self-organised patterns and structures, materials, phase transitions)
  - Modelling social interaction!

People

- Huge area, lots of people involved but..
  - John von Neumann considered the pioneer of CA's due to his early work on self-reproducing machines
  - John Conway discovered the "game of life" rules to explore life-like properties that emerge
  - Stephen Wolfram has explored CA extensively and argues that they offer a new kind of science free of the constraints and limitations of mathematics (is the universe a big CA?)
  - Chris Langton explored CA and also produced a self-reproducing system (Langton's Loop) and was a founder of the area of "artificial life" (we will look at Alife later)
- CA's appear to bring together computation, biology and physics in the search for (general) principles of complexity and self-organisation

John von Neumann

- 1903 – 1957, Mathematician
- Introduced CA’s by 1940’s
- Interested in exploring self-reproducing machines and used CA to define such a universal machine
- Applied his ideas to many areas: computers, economics, explosions, gambling, maths
- Anti-communist and anti-fascist became major player in US military-industrial complex
- Helped create A-bomb in Manhattan Project
- Interesting character…

John von Neumann

- Crashed cars a lot
- Consulted for CIA, RAND
- Played loud music in the office
- Involved in formulation of the MAD Cold War strategy
- Co-invented game theory
- Oppenheimer said physicists involved in the Manhattan project had "known sin”. Neumann’s response was that "sometimes someone confesses a sin in order to take credit for it.”
- *Good Wikipedia page on him (as of 30/8/2014)*
- *Good documentary: http://youtu.be/VTS9O0CoVng*

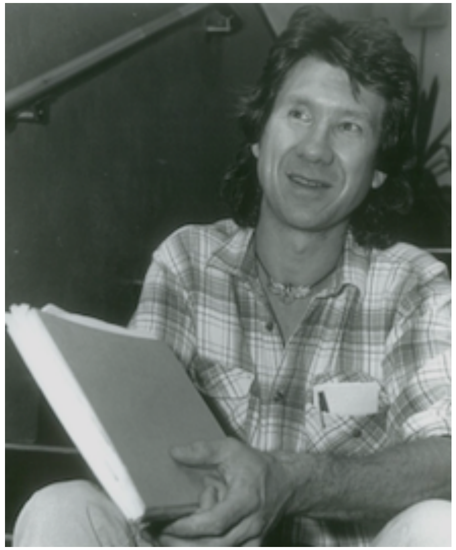
John von Neumann Quotes

- “Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin.”
- “The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work.”
- “It is just as foolish to complain that people are selfish and treacherous as it is to complain that the magnetic field does not increase unless the electric field has a curl. Both are laws of nature.”
- *Sourced from wikiquote (so might be not true!)*

Stephen Wolfram

- Physics background / entrepreneur
- co-creator of Mathematica software
- Did work on CA's from the early 80's
- Produced book "New Kind of Science" (NKS) in 2002
- Argues that laws of the universe could be CA rules – existing maths and physics not best way to model universe
- Some have argued that NKS is not good scholarship because it contains few citations, makes overblown claims and is self-published, others that Wolfram is a genius
- One might view his ideas as "CA fundamentalist"!
- *See critical review of NKS at: http://www.ams.org/notices/200302/fea-gray.pdf*
- *NKS online: http://www.wolframscience.com/nksonline/toc.html (be careful! Be critical)*

Chris Langton



- Computer Scientist
- Founder of Artificial Life in 1980's
- Invented "Langton loops"
- Simplification of Neumann's self-reproducing CA
- Neumann's CA had large number of states (29) and required a large number of cells (thousands) but it was universal – he was interested in evolution
- Universality meant it could produce more complex or different versions of itself not just copies
- Langton's CA was not universal but a lot simpler (8 states, <100 cells)
- *C. G. Langton (1984). "Self-reproduction in cellular automata". Physica D 10: 135–144.*
- *Video of Langton Loop: http://youtu.be/2iDc4C6vbcc*

John Conway

- Mathematician
- Significant contributions to many areas
- Invented game of life over coffee discussions
- Game was described in Scientific American (1970) for fun
- This made it popular and introduced many people to CA
- In recent interview he said he is a little sick of being only associated with the game which for him was just a bit of fun, not his serious work (I've done just that!)
- Two interesting interviews: http://youtu.be/R9Plq-D1gEk and http://youtu.be/E8kUJL04ELA
- Original Sci. Am. Article: Gardner, Martin (October 1970). Mathematical Games – The fantastic combinations of John Conway's new solitaire game "life”. Scientific American 223. pp. 120–123.

CA models of social phenomena

- Many variants of CA models claim to give insights into social phenomena, e.g.:
  - Racial and other kinds of Segregation (Schelling 1971)
  - Riots / Rebellion / Revolutions (Granovetter 1978, Epstein 2002)
  - Emergence of cultural groups / nation states (Axelrod 1997)
  - Emergence of empires and alliances between nation states (Axelrod 1995)
- We will look at Schelling's model in detail
- We will show how P2P researchers have used the model to propose distributed algorithms

Book chapters on CA

- Gilbert et al (2005) Chapter 7
- Flake (1998) Chapter 15
- Some questions (for fun):
  – Does game of life and 1D CA's tell us anything about social interaction?
  – Can these models inspire distributed algorithms?
  – How useful are these models? What are they modelling?